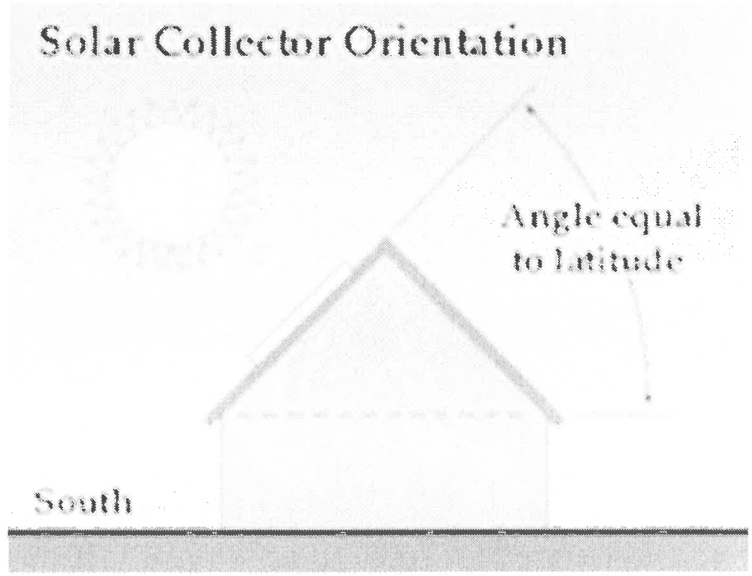
Ch. 2 Trigonometry Lessons and Practice

Introduction to Trigonometry

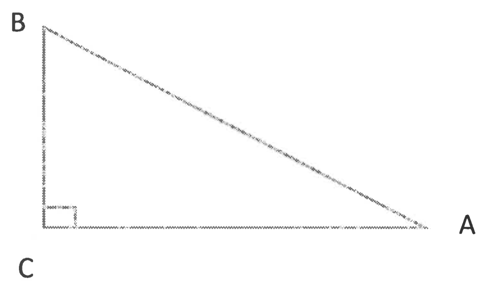
If you can never move the solar panel, place it at an angle equal to the degree of latitude.



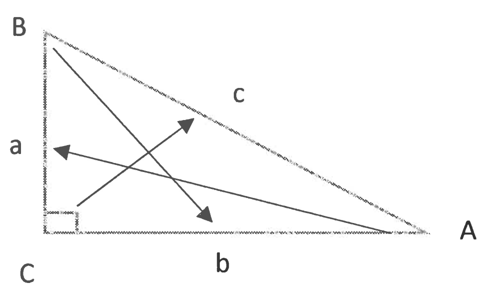
South-facing solar panels on a roof work best when the **angle of inclination** of the roof is approximately equal to the latitude of the house. The **angle of inclination** is the angle between the roof and the horizontal.

When an architect designs a house that will have solar panels on its roof, she has to determine the width and height of the roof so that the panels work efficiently.

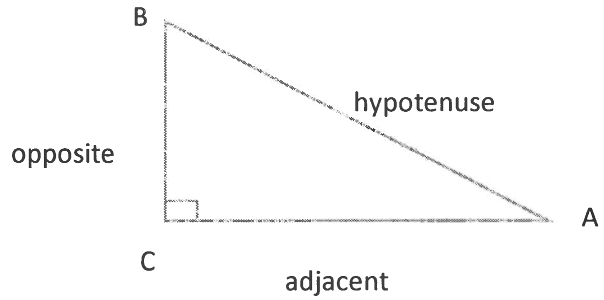
Is there an easier way to measure the angle than to use a protractor or some other measuring device? There is! It is called Trigonometry. We are going to work with only right triangles in this chapter. We name the sides of a right triangle in relation to one of its acute angles. What is an acute angle? (an angle less than 90 degrees)



But first, let's look at the letter names for the sides. Look at triangle ABC. Angles are indicated by capital letters and the sides across from them are named by lowercase letters.



We already know that the longest side in a right triangle is called the hypotenuse. The other two sides are called opposite and adjacent. But the **opposite and adjacent will change depending on which acute angle you are using as a reference point.** If we use angle A, the side across from it is the opposite, the side beside it (that isn't the hypotenuse) is the adjacent and the side across from the 90 degree angle is the hypotenuse.

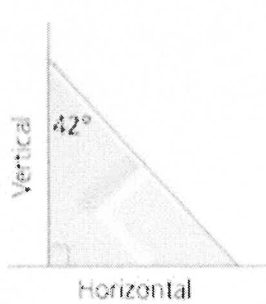
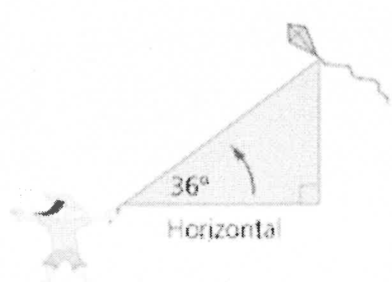
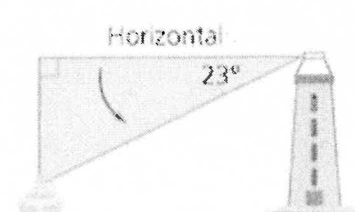


There are three primary trigonometric ratios. The ratio that we are going to learn in this section is the Tangent Ratio. It is the ratio of the opposite side over the adjacent side. We represent it as:

$$tanA = \frac{opposite}{adjacent}$$

We often express it as a decimal that compares the lengths of the sides. For example, if tanA = 1.5, that means the opposite side is 1.5 times as big as the adjacent side.

<u>We will learn about the other two ratios in a different lesson.</u>

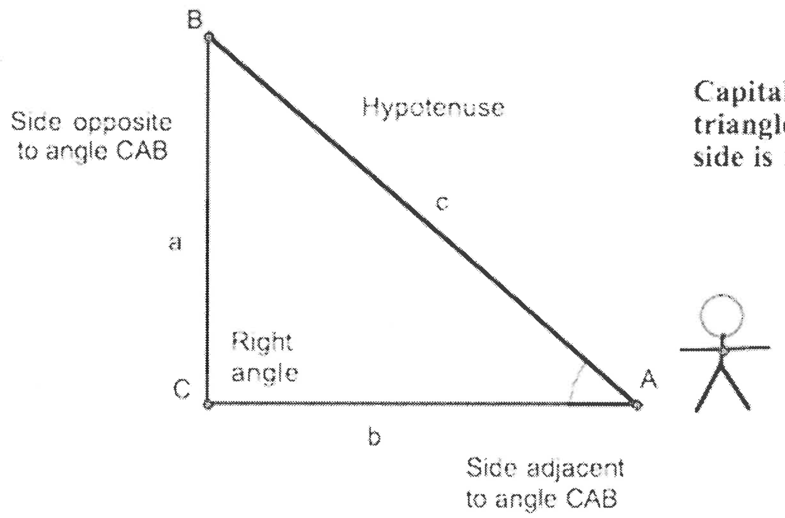
Angle of inclination	Angle of elevation	Angle of depression
Formed between the vertical or horizontal as defined by the situation. The angle of inclination with the vertical is 42°.	Formed by looking up from the horizontal The angle of elevation is 36°.	Formed by looking down from the horizontal. The angle of depression is 23°.
		

Sine, Cosine and Tangent - Three Functions, but same idea. We'll learn them separately and then put them all together.

- **Right Triangle**

Sine, Cosine and Tangent are the main functions used in Trigonometry and are based on a Right-Angled Triangle.

Before getting stuck into the functions, it helps to give a name to each side of a right triangle:



Capital letters name the vertices of the triangle. Lower case letters name the sides. A side is named after the angle opposite it.

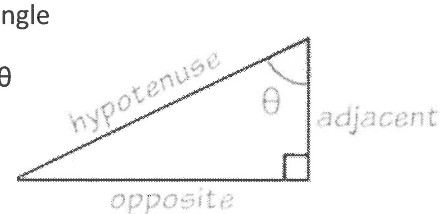
To make this easy picture yourself standing next to angle CAB. If you walk through the angle towards the right angle, you are walking on the *adjacent* side and the 'wall' you are approaching is the *opposite* side.

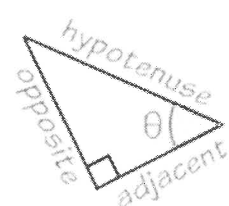
"Hypotenuse" is the long one and is **always** opposite (across from) the right angle. It doesn't change locations. (*Remember hypotenuse from Pythagoras' theorem*?)

The adjacent and opposite **change** depending on which angle you're interested in (which we often label with the Greek letter, "theta", θ.*)*

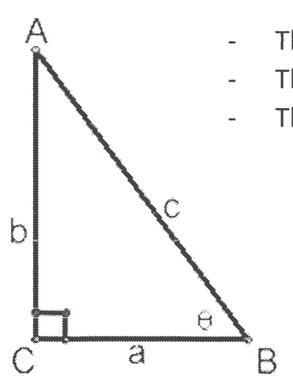
"Opposite" is opposite to (**across from**) the angle

"Adjacent" is adjacent (**next to**) to the angle θ





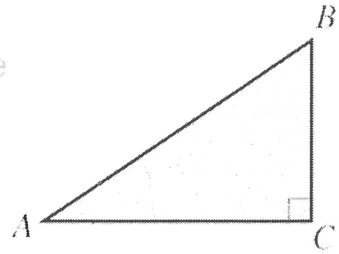
Try it: Identify:



- The side adjacent to θ _____
- The side opposite to θ _____
- The hypotenuse ______

A little bit of review.. finding missing angles, missing sides

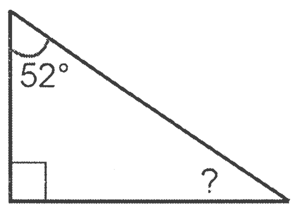
- A useful and time-saving fact about right triangles is that the sum of the acute of any right triangle is 90°.



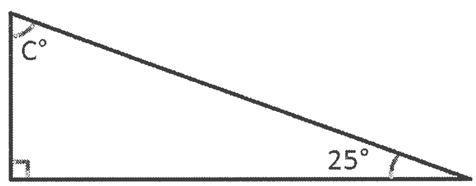
$$A + B = 90°$$

- We know that the sum of the interior angles of any triangle is 180°. A right triangle, by definition, contains a right angle, whose measure is 90°. That leaves 90° to be divided between the two acute angles.

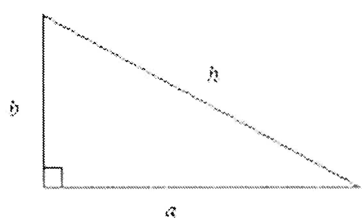
a) ? = ______________



b) c = _________



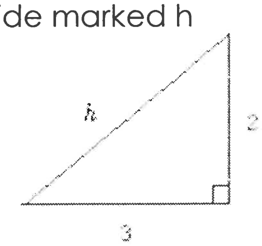
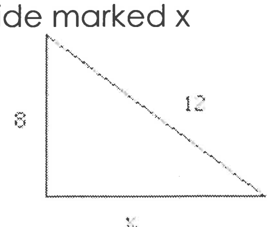
The **Theorem of Pythagoras** states that in a <u>right-angled triangle</u> the **square** of the length of the **hypotenuse** (the longest side) is equal to the **sum** of the **squares** of the lengths of the **two other sides** (the two shorter sides that form the right angle).



$$a^2 + b^2 = h^2$$

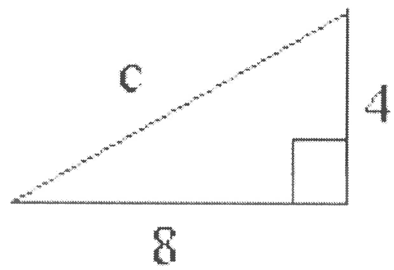
The hypotenuse, h, is always opposite the right angle.

The key to remember is that "**h**" is <u>always the hypotenuse</u> in this equation. The other 2 sides are transitive, so <u>either</u> side can be called "**a**" or "**b**".

Examples	Answers
Find the length of the side marked h 	$h^2 = 3^2 + 2^2$ $h^2 = 9 + 4$ $h^2 = 13$ $h = \sqrt{13}$
Find the length of the side marked x 	$12^2 = 8^2 + x^2$ $144 = 64 + x^2$ $144 - 64 = x^2$ $80 = x^2$ $\sqrt{80} = x$

Find the length of the third side of each right triangle, using Pythagoras. Show all work algebraically. Represent the length first as an entire radical and then write it as a decimal truncated (- chopped, not rounded) to 4 decimal places. When we are working on a problem and we need to round in the middle of a solution, we truncate to 4 decimal places. We only round (if needed) in the FINAL solution to the precision indicated.

3.



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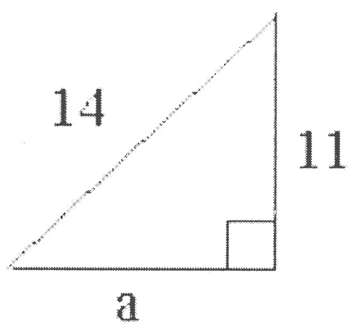

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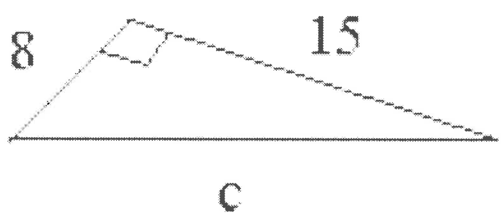
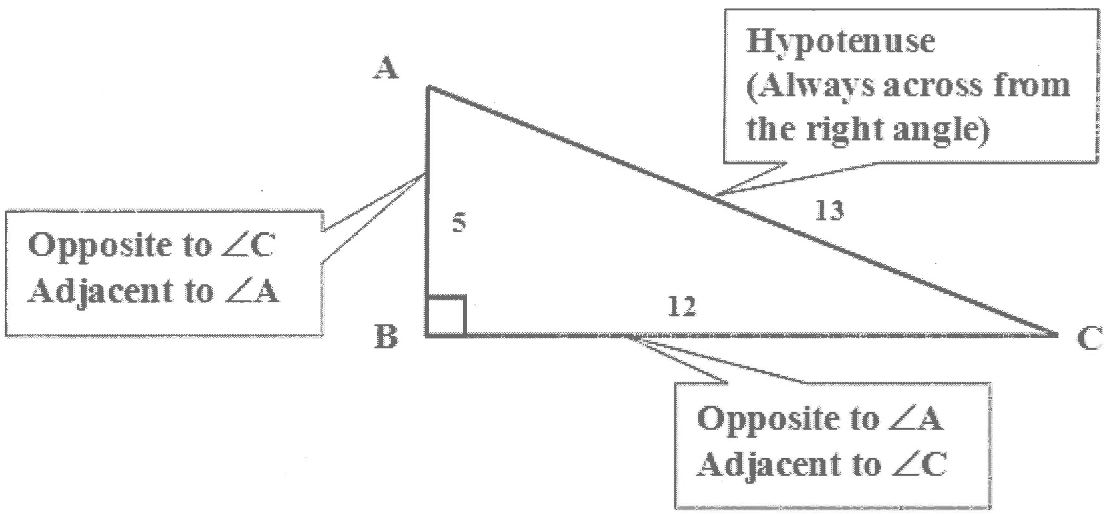


2.1 The Tangent Ratio

LESSON FOCUS: Develop the tangent ratio and relate it to the angle of inclination of a line segment.

Trigonometry is the study of the relationships among the **sides and angles** of triangles. One such relationship is the **tangent ratio**, which is an example of a trigonometric ratio.



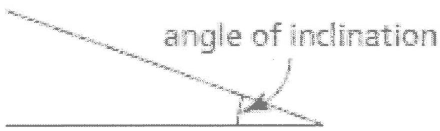
Definition: $\text{tangent ratio} = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

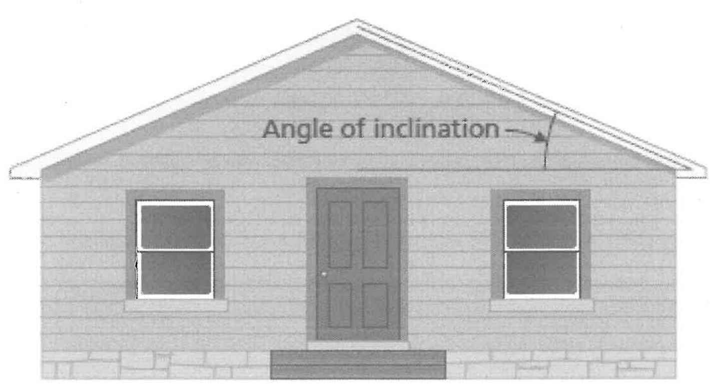
Short hand:

Angle

$$\underbrace{\tan\theta = \frac{\text{opp}}{\text{adj}}}_{\textbf{Ratio}}$$

The **angle of inclination** of a line or line segment is the acute angle it makes with the horizontal.





Knowing Your Calculator

Find the following to 3 decimal places.

a) tan 27°

b) tan 72°

Find ∠H to the nearest degree.

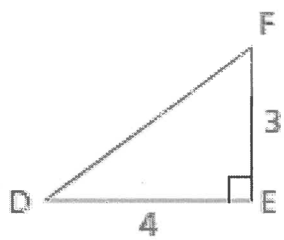
a) tan H = 4.332

b) tan H = 0.651

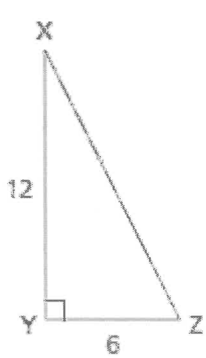
c) tan H = $\frac{3}{4}$

d) tan H = $\frac{9}{5}$

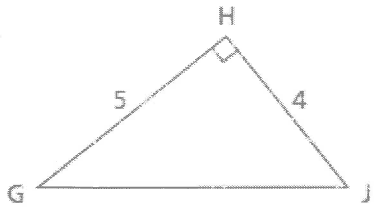
Example 1: Determining the Tangent Ratios for Angles
Determine tan D and tan F.



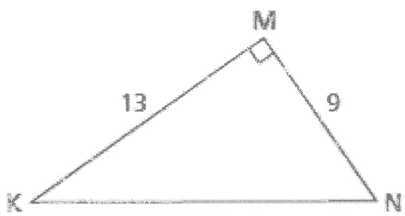
CHECK YOUR UNDERSTANDING
Determine tan X and tan Z. [Answer: tan X = 0.5; tan Z = 2]



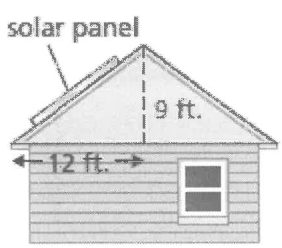
Example 2: Using the Tangent Ratio to Determine the Measure of an Angle
Determine the measures of $\angle G$ and $\angle J$ to the nearest tenth of a degree.



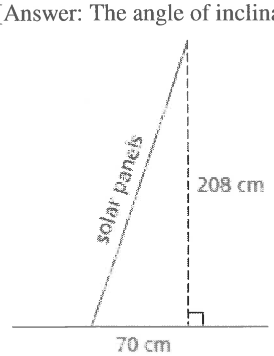
CHECK YOUR UNDERSTANDING
Determine the measures of $\angle K$ and $\angle N$ to the nearest tenth of a degree.
[Answer: $\angle$ K□ 34.7°; $\angle$ N□ 55.3°]



Example 3: Using the Tangent Ratio to Determine an Angle of Inclination
The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is the best for Fort Smith. Justify your answer.



CHECK YOUR UNDERSTANDING
Clyde River on Baffin Island, Nunavut, has a latitude of approximately 70°. The diagram shows the side view of some solar panels. Determine whether this design for solar panels is the best for Clyde River. Justify your answer.
[Answer: The angle of inclination is approximately 71°. So, the design is the best.]



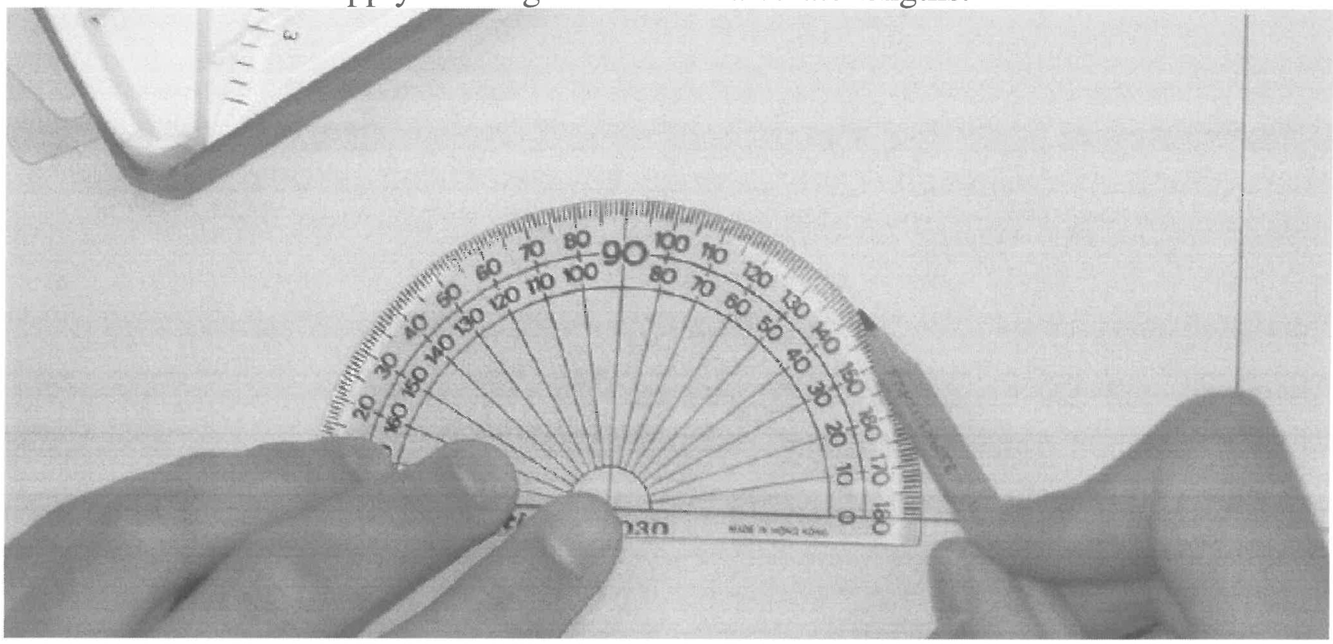
Example 4: Using the Tangent Ratio to Solve a Problem
A 10-ft. ladder leans against the side of a building with its base 4 ft. from the wall. What angle, to the nearest degree, does the ladder make with the ground?

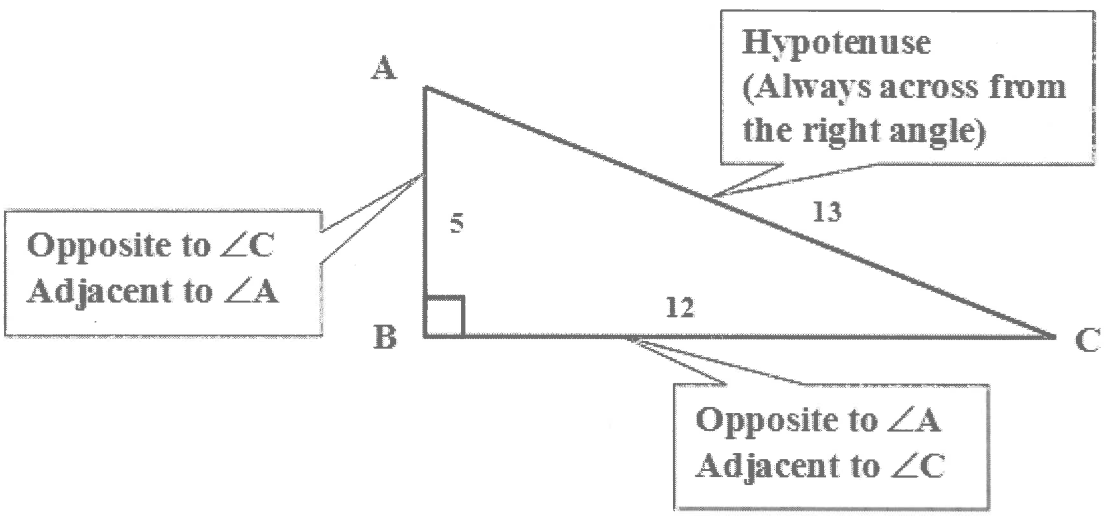
CHECK YOUR UNDERSTANDING
A support cable is anchored to the ground 5 m from the base of a telephone pole. The cable is 19 m long. It is attached near the top of the pole. What angle, to the nearest degree, does the cable make with the ground?
[Answer: The angle is approximately 75°.]

Homework: Page 75 #3-6, 8, 10, 11, 13-16, 19, 20

2.2 Using the Tangent Ratio to Calculate Lengths

LESSON FOCUS: Apply the tangent ratio to calculate lengths.





Definition: $\text{tangent ratio} = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

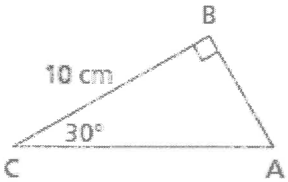
Short hand:

Angle

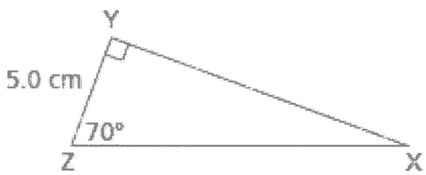
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

Ratio

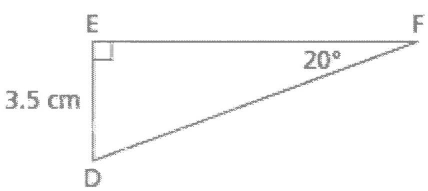
Example 1: Determining the Length of a Side Opposite a Given Angle
Determine the length of AB to the nearest tenth of a centimetre.



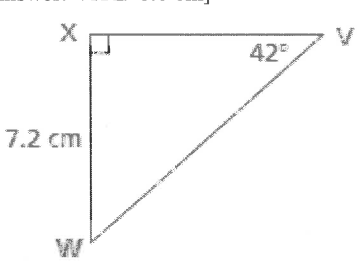
CHECK YOUR UNDERSTANDING
Determine the length of XY to the nearest tenth of a centimetre. [Answer: XY□ 13.7 cm]



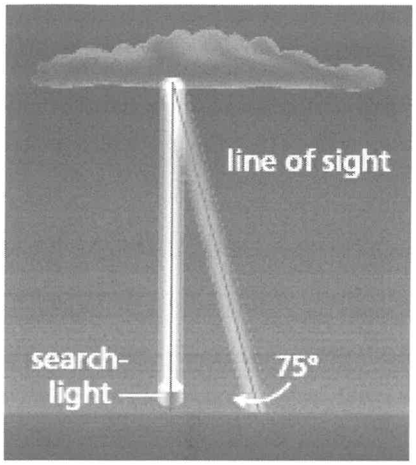
Example 2: Determining the Length of a Side Adjacent to a Given Angle
Determine the length of EF to the nearest tenth of a centimetre.



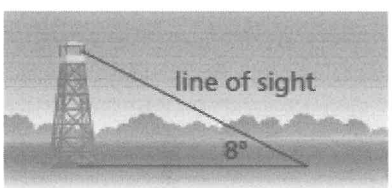
CHECK YOUR UNDERSTANDING
Determine the length of VX to the nearest tenth of a centimetre.
[Answer: VX□ 8.0 cm]



Example 3: Using Tangent to Solve an Indirect Measurement Problem
A searchlight beam shines vertically on a cloud. At a horizontal distance of 250 m from the searchlight, the angle between the ground and the line of sight to the cloud is 75°. Determine the height of the cloud to the nearest metre.



CHECK YOUR UNDERSTANDING
At a horizontal distance of 200 m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 8°. How high is the tower to the nearest metre? The diagram is *not* drawn to scale.
[Answer: 28 m]



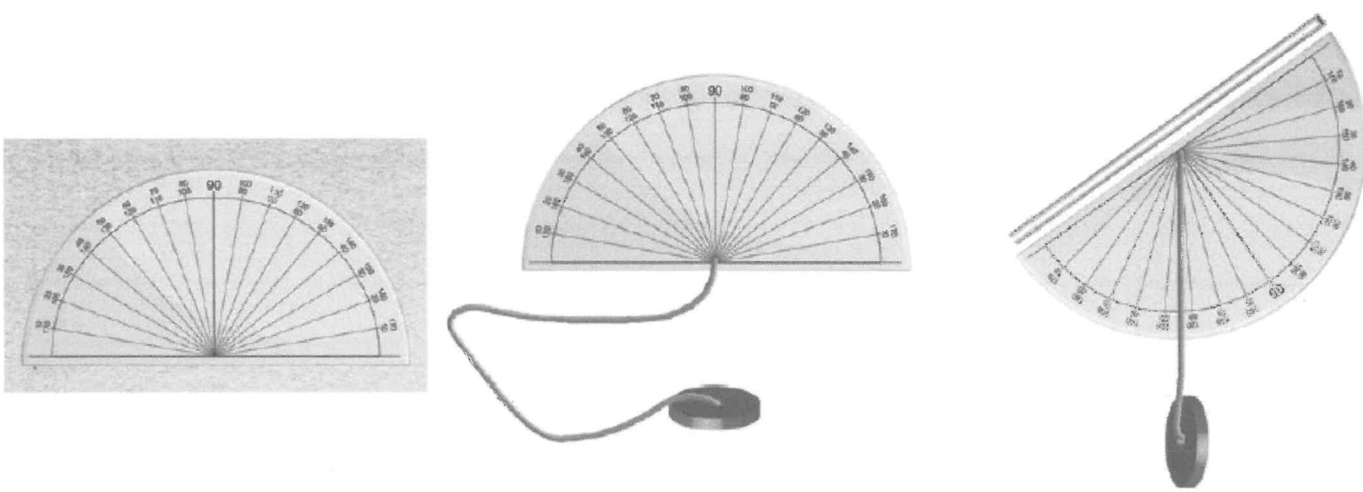
Homework: Page 82 #3–13

2.3 Math Lab: Measuring an Inaccessible Height

LESSON FOCUS: Determine a height that cannot be measured directly.

A. Make a drinking straw clinometer:

- Glue or tape the paper protractor to the cardboard. Carefully cut it out.
- Use the needle to pull the thread through the cardboard at the centre of baseline of the protractor. Secure the thread to the back of the cardboard with tape. Attach the weight to the other end of the thread.
- Tape the drinking straw along the baseline of the protractor for use as a sighting tube.



How to Measure A Tall Building or Skyscraper Without Leaving the Ground

The method of measurement requires a protractor (clinometer), a straw and a measuring tape. This method does not use shadows. Instead it uses the accurate visual senses of the measurer. The protractor should be as large as possible for an accurate angle observation and for further calculation. The pictures attached describe the procedure to make your own clinometer. Just select a protractor with a small hole (usually provided at the origin) and tape a straw at its flat end. Also knot a string onto the protractor with a small weight at its other end. Now we can use this simple device to look at the building top to measure its height.

Procedure:

Ensure the straw is clear and try to locate the building top through the straw. The angle of elevation is evident from the string which always acts downwards because of the pull of the weight due to gravity.

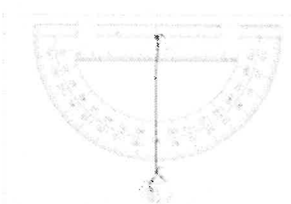
Note down the distance between the building and the point at which the building top is viewed through the straw. Also note the angle from the clinometer. The height of the building is calculated by using the formula:

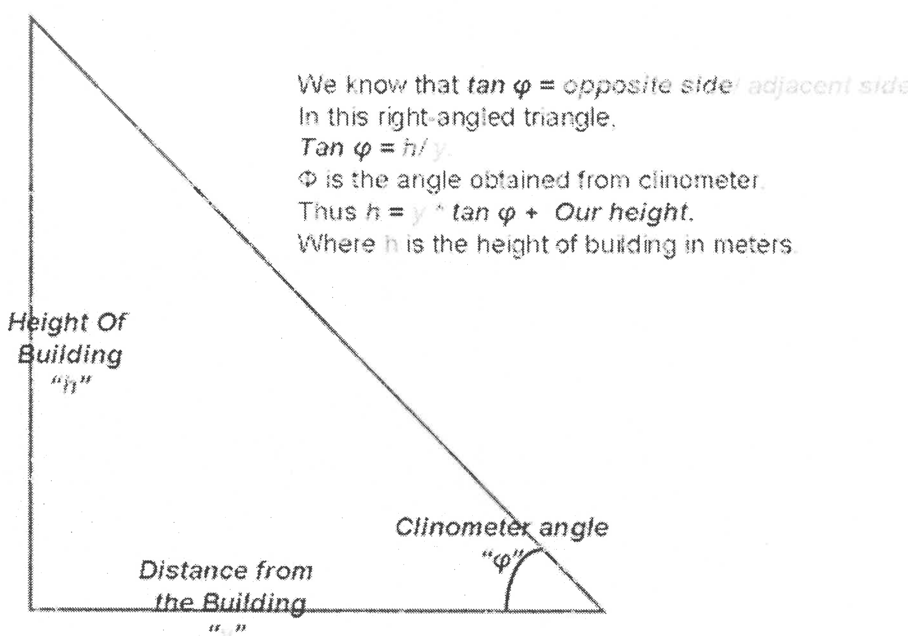
Height of the building = y * tan x + measurer's height.

Where

y = distance of the measurer from the building.

x = the angle measured from the clinometer.



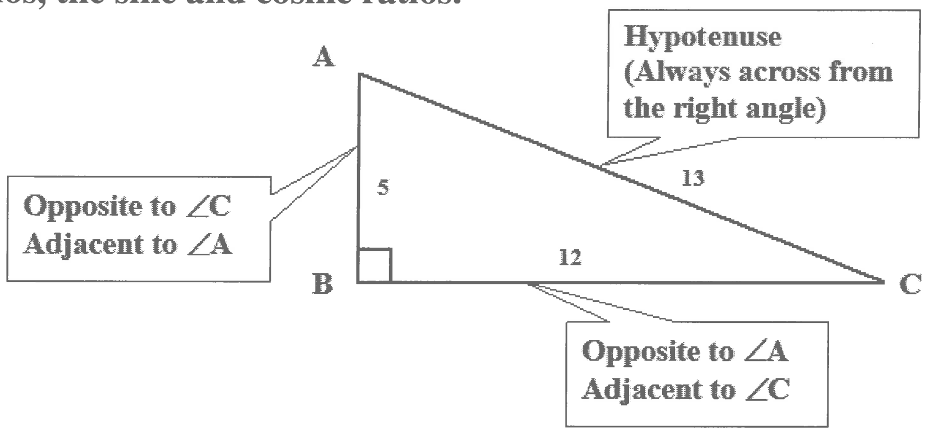


Text work p.88 #1-5

2.4 The Sine and Cosine Ratio

LESSON FOCUS: Develop and apply the sine and cosine ratios to determine angle measures.

Last class we look at the tangent ratio, today we look at the other two trigonometric ratios, the sine and cosine ratios.



The Sine Ratio

$$\underbrace{\overset{\text{angle}}{\sin\theta}}_{\text{Ratio}} = \underbrace{\frac{opp}{hyp}}$$

The Cosine Ratio

$$\underbrace{\overset{\text{angle}}{\cos\theta}}_{\text{Ratio}} = \underbrace{\frac{adj}{hyp}}$$

Knowing Your Calculator

Find the following to 3 decimal places.

a) sin 27°

b) cos 72°

Find ∠H to the nearest degree.

a) sin H = 0.332

b) cos H = 0.651

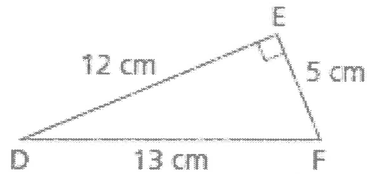
c) $\sin H = \frac{3}{4}$

d) $\cos H = \frac{3}{7}$

Example 1: Determining the Sine and Cosine of an Angle

a) In □ DEF, identify the side opposite ∠D and the side adjacent to ∠D.

b) Determine sin D and cos D to the nearest hundredth.

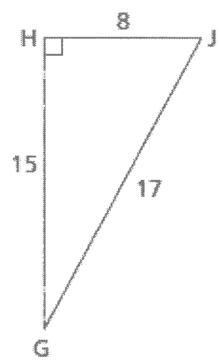


CHECK YOUR UNDERSTANDING

[Answers: a) HJ, HG, b) sin G□ 0.47; cos G□ 0.88]

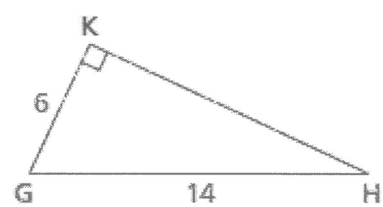
a) In □ GHJ, identify the side opposite ∠G and the side adjacent to ∠G.

b) Determine sin G and cos G to the nearest hundredth.



Example 2: Using Sine or Cosine to Determine the Measure of an Angle

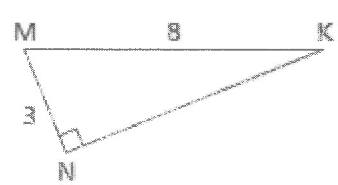
Determine the measures of ∠G and ∠H to the nearest tenth of a degree.



CHECK YOUR UNDERSTANDING

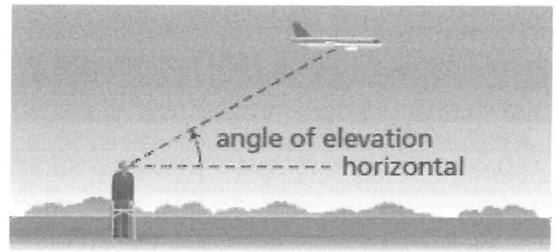
Determine the measures of ∠K and ∠M to the nearest tenth of a degree.

[Answer: ∠K□ 22.0°, ∠M□ 68.0°]



Terms you need to know:

- **Angle of inclination:** **The *angle of inclination* of a line is the angle made between the line and the horizontal line which it intersects.**
- **Angle of elevation:** **The line of sight made with the horizontal as you up towards an object.**



- **Angle of depression:** **The line of sight made with the horizontal as you down towards an object.**

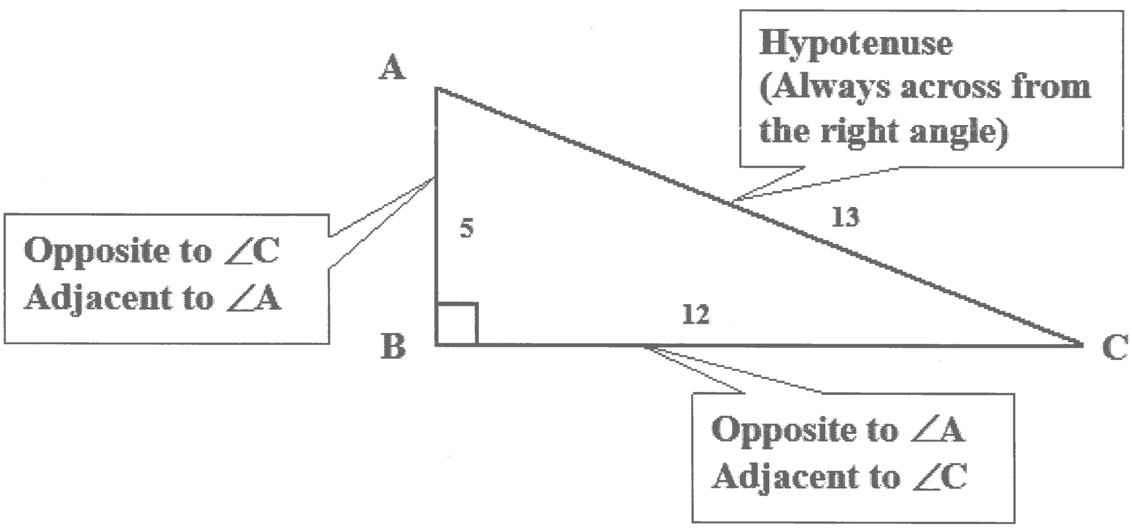
Example 3: Using Sine or Cosine to Solve a Problem
A water bomber is flying at an altitude of 5000 ft. The plane's radar shows that it is 8000 ft. from the target site. What is the **angle of elevation** of the plane measured from the target site, to the nearest degree?

CHECK YOUR UNDERSTANDING
An observer is sitting on a dock watching a float plane in Vancouver harbour. At a certain time, the plane is 300 m above the water and 430 m from the observer. Determine the angle of elevation of the plane measured from the observer, to the nearest degree.
[Answer: approximately 44°]

Homework: Page 95 #4–6, 9–15

2.5 Using the Sine and Cosine Ratios to Calculate Lengths

LESSON FOCUS: Use the sine and cosine ratios to determine lengths indirectly.



The Sine Ratio

angle

$$\underbrace{\sin\theta}_{\text{Ratio}} = \underbrace{\frac{opp}{hyp}}$$

Ratio

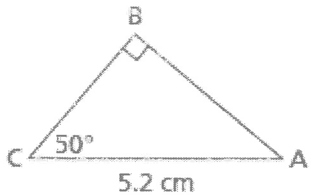
The Cosine Ratio

angle

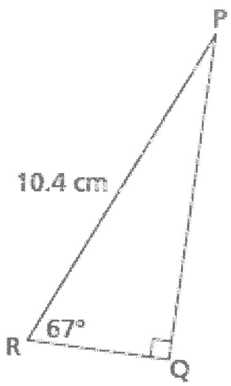
$$\underbrace{\cos\theta}_{\text{Ratio}} = \underbrace{\frac{adj}{hyp}}$$

Ratio

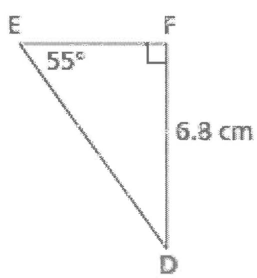
Example 1: Using the Sine or Cosine Ratio to Determine the Length of a Leg
Determine the length of BC to the nearest tenth of a centimetre.



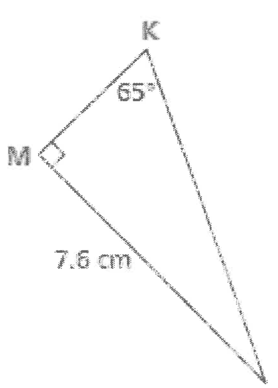
CHECK YOUR UNDERSTANDING
Determine the length of PQ to the nearest tenth of a centimetre. [Answer: PQ ≈ 9.6 cm]



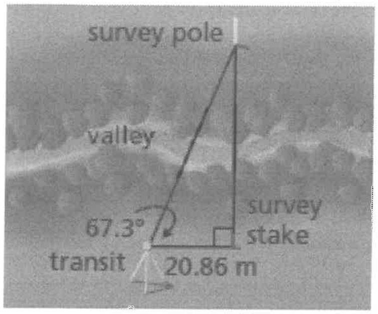
Example 2: Using Sine or Cosine to Determine the Length of the Hypotenuse
Determine the length of DE to the nearest tenth of a centimetre.



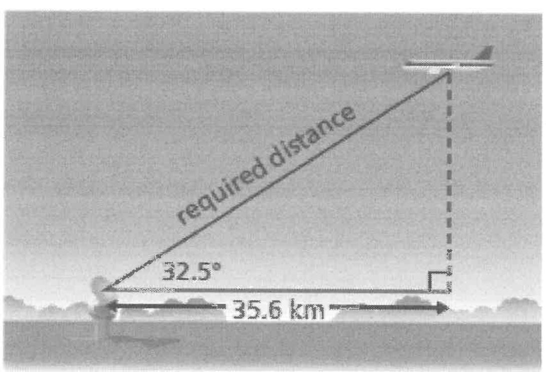
CHECK YOUR UNDERSTANDING
Determine the length of JK to the nearest tenth of a centimetre. [Answer: JK ≈ 8.4 cm]



Example 3: Solving an Indirect Measurement Problem
A surveyor made the measurements shown in the diagram. How could the surveyor determine the distance from the transit to the survey pole to the nearest hundredth of a metre?



CHECK YOUR UNDERSTANDING
From a radar station, the angle of elevation of an approaching airplane is 32.5°. The horizontal distance between the plane and the radar station is 35.6 km. How far is the plane from the radar station to the nearest tenth of a kilometre? [Answer: 42.2 km]



Homework: Page 101 #4–11

2.6 Applying the Trigonometric Ratios

LESSON FOCUS: Use a primary trigonometric ratio to solve a problem modeled by a right triangle.

> **Solving a triangle** means to determine the measures of all the angles and the lengths of all the sides in the triangle.

The Sine Ratio

angle

$$\sin\theta = \frac{opp}{hyp}$$

Ratio

The Cosine Ratio

angle

$$\cos\theta = \frac{adj}{hyp}$$

Ratio

The Tangent Ratio

angle

$$\tan\theta = \frac{opp}{adj}$$

Ratio

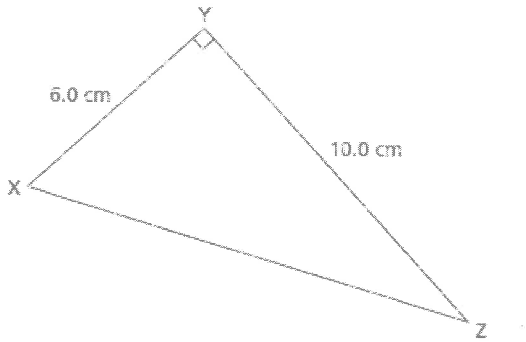
S	O	H	C	A	H	T	O	A
I	P	Y	O	D	Y	A	P	D
N	P	P	S	J	P	N	P	J
E	O	O	I	A	O	G	O	A
	S	T	N	C	T	E	S	C
	I	E	E	E	E	N	I	E
	T	N		N	N	T	T	N
	E	U		T	U		E	T
		S			S			
		E			E			

To solve a right triangle means to find all the unknown sides and unknown angles. This can be done if you know either at least:

- **two sides, or**
- **one angle (other than the right angle) and one side**

Example 1: Solving a Right Triangle Given Two Sides

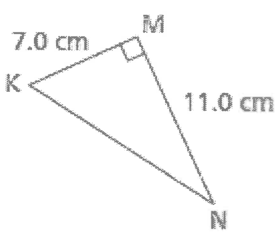
Solve □ XYZ. Give the measures to the nearest tenth.



CHECK YOUR UNDERSTANDING

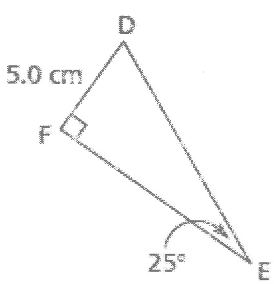
Solve this triangle. Give the measures to the nearest tenth.

[Answers: KN□ 13.0 cm; ∠K□ 57.5°; ∠N□ 32.5°]



Example 2: Solving a Right Triangle Given One Side and One Acute Angle

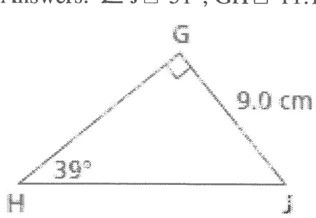
Solve this triangle. Give the measures to the nearest tenth where necessary.



CHECK YOUR UNDERSTANDING

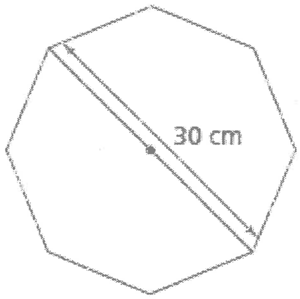
Solve this triangle. Give the measures to the nearest tenth where necessary.

[Answers: ∠J□ 51°; GH□ 11.1 cm; HJ□ 14.3 cm]



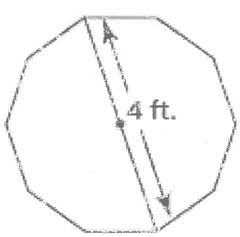
Example 3: Solving a Problem Using the Trigonometric Ratios

A small table has the shape of a regular octagon. The distance from one vertex to the opposite vertex, measured through the centre of the table, is approximately 30 cm. There is a strip of wood veneer around the edge of the table. What is the length of this veneer to the nearest centimetre?



CHECK YOUR UNDERSTANDING

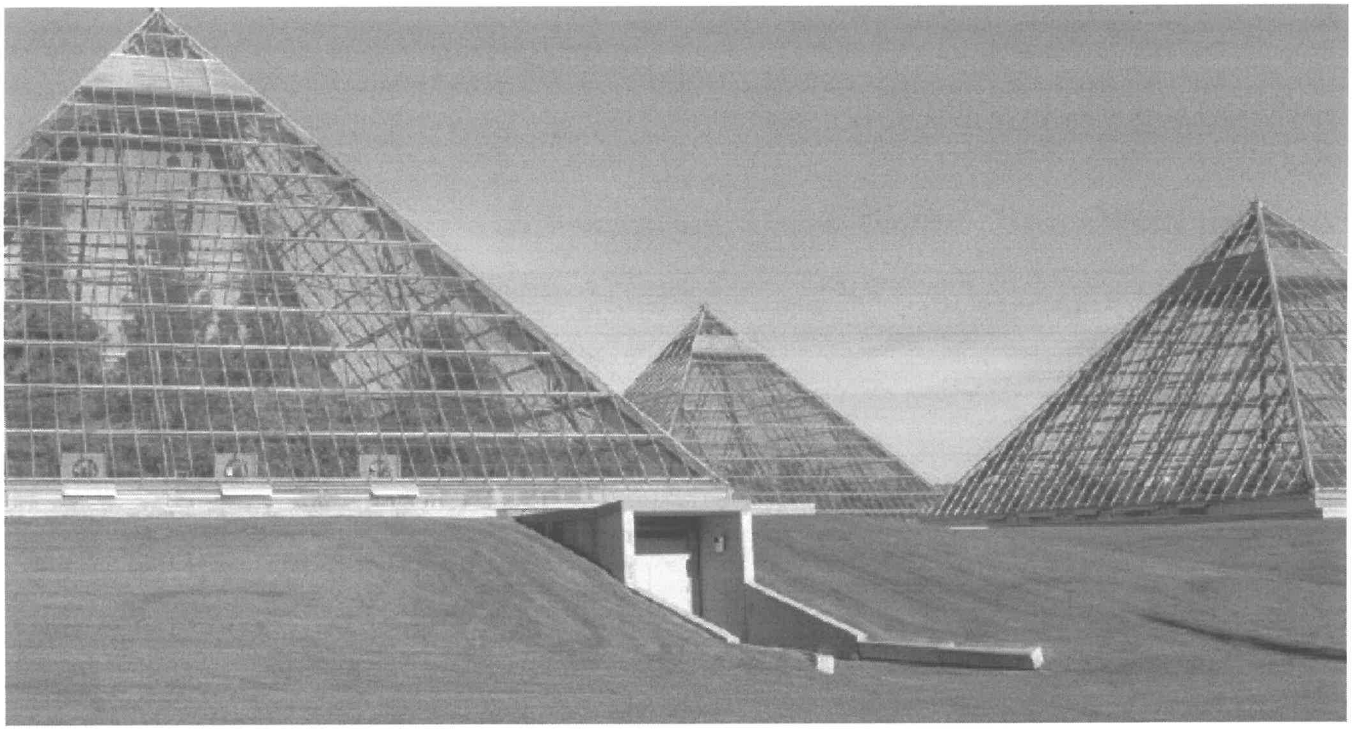
A window has the shape of a regular decagon. The distance from one vertex to the opposite vertex, measured through the centre of the window, is approximately 4 ft. Determine the length of the wood molding material that forms the frame of the window, to the nearest foot. [Answer: approximately 12 ft.]



Homework: Page 111 #4–12

2.7 Solving Problems Involving More than One Right Triangle

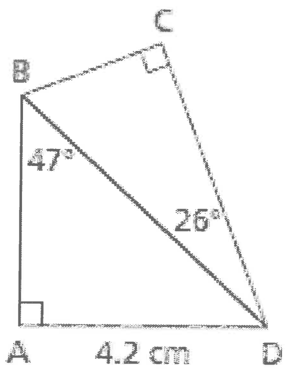
LESSON FOCUS: Use trigonometry to solve problems modeled by more than one right triangle.



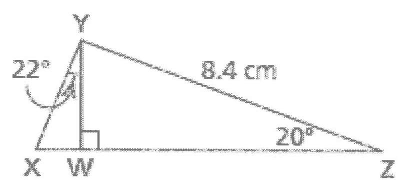
Make Connections

The Muttart Conservatory in Edmonton has four climate-controlled square pyramids, each representing a different climatic zone. Each of the tropical and temperate pyramids is 24 m high and the side length of its base is 26 m. How do you think the architects determined the angles at which to cut the glass pieces for each face at the apex of the pyramid?

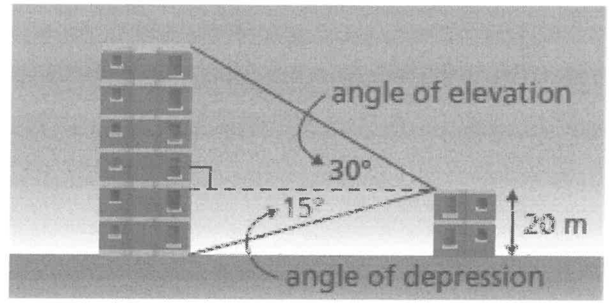
Example 1: Calculating a Side Length Using More than One Triangle
Calculate the length of CD to the nearest tenth of a centimetre.



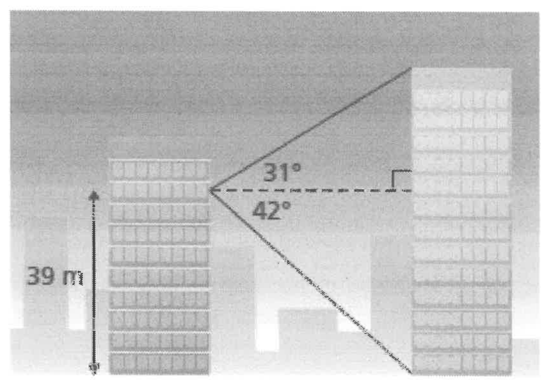
CHECK YOUR UNDERSTANDING
Calculate the length of XY to the nearest tenth of a centimetre. [Answer: XY□ 3.1 cm]



Example 2: Solving a Problem with Triangles in the Same Plane
From the top of a 20-m high building, a surveyor measured the angle of elevation of the top of another building and the **angle of depression** of the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.

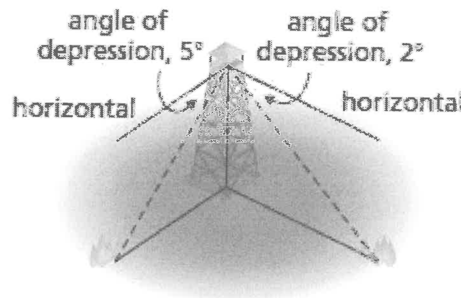


CHECK YOUR UNDERSTANDING
A surveyor stands at a window on the 9th floor of an office tower. He uses a clinometer to measure the angles of elevation and depression of the top and the base of a taller building. The surveyor sketches this plan of his measurements. Determine the height of the taller building to the nearest tenth of a metre. [Answer: Approximately 65.0 m]



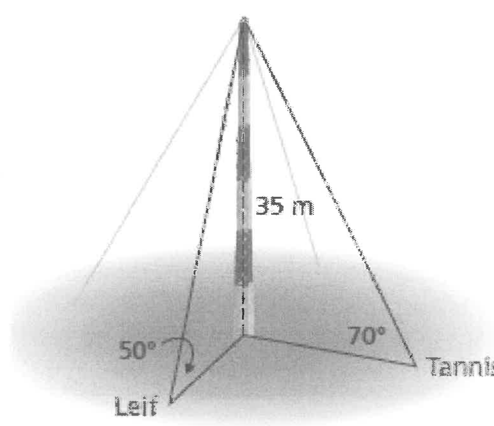
Example 3: Solving a Problem with Triangles in Different Planes

From the top of a 90-ft. observation tower, a fire ranger observes one fire due west of the tower at an angle of depression of 5°, and another fire due south of the tower at an angle of depression of 2°. How far apart are the fires to the nearest foot? The diagram is *not* drawn to scale.



CHECK YOUR UNDERSTANDING

A communications tower is 35 m tall. From a point due north of the tower, Tannis measures the angle of elevation of the top of the tower as 70°. Her brother Leif, who is due east of the tower, measures the angle of elevation of the top of the tower as 50°. How far apart are the students to the nearest metre? The diagram is *not* drawn to scale.
[Answer: About 32 m]



Homework: Page 118 #4–6, 8–14